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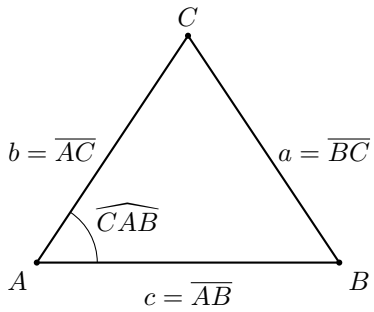
ÉCOLE POLYTECHNIQUE DE LOUVAIN

**UCLouvain - Admission Exam – Trigonometry – Monday, August 31, 2026**

Duration: 1h30 – 4 pages – answer directly within the boxes – no calculators

## Notations and conventions

- $\sin^n x$ means $(\sin(x))^n$, and similarly $\cos^n x = (\cos(x))^n$, etc.
- Segment lengths are denoted with a bar: $\overline{AB}$ denotes the distance between points $A$ and $B$.
- An angle such as $\widehat{BAC}$ denotes the angle formed at point $A$, between segments $[AB]$ and $[AC]$.
- In a triangle $ABC$, the lengths $a$, $b$, $c$ denote respectively the sides opposite to vertices $A$, $B$, and $C$.
- Angles are expressed in radians unless indicated otherwise with a degree symbol. For example: $15°$.




## General Instructions

- No calculators or connected devices allowed.
- The exam consists of 4 **single-sided** pages on which you are asked to answer **3 questions**.
- The reverse sides are not scanned but may be used as scratch paper.
- Please **do not unstaple** the sheets.
- Write your last name, first name, and registration number on all 4 pages.
- The use of personal scratch paper is strictly forbidden.
- You may use a pencil, but ensure that your answers are clearly legible. More generally, provide as neat a paper as possible.
- At the end of the examination, please hand in all sheets in order. Do not hand in any additional scratch paper.

**Question 1 – Trigonometric equation (≈ 8 points)**

Find all real solutions to the following trigonometric equation

$$\sin^2(4x) + \cos^4(2x) = 1.$$

Scan Q1

**Question 2 – Circle inscribed in a circular sector (≈ 6 points)**

**2a** A circular sector of radius $R$ and central angle $\theta$ (with $0 < \theta < \pi$) is given. A circle of radius $r$ is inscribed inside this sector, such that it is tangent to the circular arc as well as to the two radii bounding the sector.

Represent the situation on a sketch indicating the center $O$ of the sector, the radius $R$, the central angle $\theta$, the inscribed circle of radius $r$, its center $C$, and a point of tangency $T$ with one of the radii.

Scan Q2

**2b** Express the radius $r$ of the inscribed circle in terms of the radius $R$ of the sector and the central angle $\theta$. Detail and justify the steps of your reasoning.

Scan Q3

**Final answer**: $r =$

**2c** Determine the value of the central angle $\theta$ (in radians and in degrees) for which the radius of the inscribed circle $r$ is equal to one third of the radius of the sector $R$ (that is, $\frac{r}{R} = \frac{1}{3}$). Justify your calculations.

Scan Q4

**Final answer**: $\theta =$

**Question 3 – Miscellaneous questions (≈ 6 points).**

Use scratch paper to work out the final answers to these 6 questions and write **only these final answers in the boxes**.

**3a** Let $ABCD$ be an isosceles trapezoid whose parallel bases $[AB]$ and $[CD]$ have lengths $\overline{AB} = x$ and $\overline{CD} = y$, respectively. The diagonals $[AC]$ and $[BD]$ intersect at a right angle (90°).

Determine the area of the trapezoid $ABCD$ in terms of $x$ and $y$.

Area = 

Scan Q5

**3b** Give only the solutions to the following equation for $0 \leq x < 2\pi$.

$\sin(2x) = \cos x$, Solution(s) = 

Scan Q6

**3c** Calculate the exact value.

$\cos^4(22.5°) - \sin^4(22.5°) =$ 

Scan Q7

**3d** If $\sin(\theta) = x$ with $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, express $\cos(3\theta)$ in terms of $x$ only. The final answer must not contain any trigonometric functions.

$\cos(3\theta) =$ 

Scan Q8

**3e** In an isosceles right triangle, let $r$ and $R$ denote the radii of the inscribed circle and circumscribed circle, respectively. Calculate the exact ratio $r/R$.

$\frac{r}{R} =$ 

Scan Q9

**3f** Calculate exactly.

$\cos(20°)\cos(40°)\cos(80°) =$ 

Scan Q10